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Reliability Estimation of Cascade System Using Rayleigh Distributions and Failure Model

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Abstract

The paper examines an n-cascade system in which components can fail in specific ways. To estimate the reliability of the cascade system, we analyze a model where the operation of an active component is influenced by m stresses that fall within an interval (a_j,b_j). A component is considered to have failed if any of the stresses exceed these defined limits. In particular, when the stress-strength of the components follows a Rayleigh distribution, we derive expressions for the reliability of the model. Additionally, we present numerical values of reliability in tabular form for selected parameter values.

Keywords: *Cascade system, Stress- Strength, reliability, Rayleigh distribution, failure.*

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Introduction

An n-cascade system is a specific type of n-standby system [6]. In a cascade system, the stresses on subsequent components are reduced by a factor k, known as the attenuation factor. This factor is typically assumed to be constant across all components or to have different fixed values for individual components. However, it is also possible for the attenuation factor to be a random variable [7]. Let $X_1, X_2,\ldots,X_n$ be the strengths of n-components in the order of activation and let $Y_1, Y_2,\ldots,Y_n$ are the stresses working on them. In cascade system after every failure the stress is modified by a factor k [1] such that

$Y_2=kY_1$, $Y_3=kY_2=k^2Y_1\ldots,Y_i=k^{i-1}Y_1$ etc.

In the stress-strength model, the reliability R of a component (or system) is defined as the probability that its strength X is greater than or equal to the stress Y applied to it, where both X and Y are random variables.

i.e. $R =\Pr (X\geq Y)$

In many scenarios, a component can fail in various ways, such as electrical or mechanical failures. Each failure type can be linked to different stresses, which can be modeled using distinct random variables. Suppose a component may fail in m different ways, each associated with a specific stress. If X represents a random variable for the strength of the component, the reliability of the component can be expressed as follows:

$$R= P[X \geq Y_1, X \geq Y_2,\ldots, X \geq Y_m] \tag{1.1}$$

The paper examines an n-cascade system in which components can fail in various ways. We focus on the following model:

Model: In an n-cascade system, we assume that the operation of an active component requires that the m stresses acting on it remain within the interval (a_j, b_j) for $j=1,2,\ldots,m$. The component will fail if any of the stresses exceed these specified limits. Both limits a_j and b_j are considered constants, and all components are assumed to be identical. For this model the constants (a_j, b_j) and (a_{ij}, b_{ij}) must have been fixed on the basis of the strength of the components.

Sriwastav and Dutta [8] examined an n-standby system incorporating different types of failures within the stress-strength model. Doloi and Gogoi [2] discussed a cascade system with Rama distributed random variables. They estimated the cascade reliability when stress strength is Rama distributed random variables. For multicomponent systems, Hilton and Fergen [4] addressed structural reliability problems where the failure modes of the components are independent. Moser and Kinser [5] explored a similar issue. Heller and Donat [3] evaluated the reliability of a "multiple-load-path" structure, which can continue functioning with m-1 failed components when it initially has m components. In this system, the applied stress is redistributed among the surviving components after each failure, and the system ultimately fails when all components are inoperable. They also considered statistical dependence among different failure modes.

Here, we have considered an n-cascade system. To our knowledge, no previous study has addressed a cascade model for this type of system. The objective of this paper is to estimate the system reliability R_n under a specific failure model.

The paper is organized as follows: Section 2 presents the general expression for the proposed model. In Section 3, the reliability expressions for an n-cascade system are derived when the stress-strength of the components follows a specific distribution. Subsection 3.1 focuses on the reliability expressions R_n for n<4, assuming Rayleigh distributions for the stress and strength. Section 4 provides numerical values for the reliabilities R(1), R(2), R(3) and R_3 under the proposed model in a tabular form.

Development of the Mathematical Models

Let us consider an n-cascade system operating under the influence of m different stresses. Our goal is to derive the reliability of this system under the proposed model.

Model: We have an n-cascade system. Let represent the stresses acting on the first component. We assume that, for a component to function correctly, the jth stress must lie within a specified interval, denoted as (a_j, b_j) for j=1, 2,…, m. When the first component fails, the second component experiences m stresses given by and the third component experiences m stresses and so on. We further assume that all components are identical, meaning that the intervals (a_j, b_j) are the same for each component.

Then the probability that the ith component works is given by

$$R'(i) = P\left[X_i \ge k^{i-1}Y_1, X_i \ge k^{i-1}Y_2, \ldots, X_i \ge k^{i-1}Y_m\right],\ i = 1,2,\ldots,n \tag{2.1}$$

Then, the reliability of the system is given by

$$R_n = R'(1) + \left[1-R'(1)\right]R'(2) + \left[1-R'(1)\right]\left[1-R'(2)\right]R'(3) + \ldots + \left[1-R'(1)\right]\left[1-R'(2)\right]\ldots\left[1-R'(n-1)\right]R'(n) \tag{2.2}$$

$$= R(1) + R(2) + \ldots + R(n), say \tag{2.3}$$

where R(i), i=1,2,…,n is the marginal reliability due to the i^{th} component, given by

$$R(i) = \left[1-R'(1)\right]\left[1-R'(2)\right]\ldots\left[1-R'(i-1)\right]R'(i) \tag{2.4}$$

But now,

$$\begin{aligned} R'(i) &= P\left[(a_1 < k^{i-1}Y_1 < b_1), (a_2 < k^{i-1}Y_2 < b_2), \ldots, (a_m < k^{i-1}Y_m < b_m)\right] \\ &= P\left(a_1 < k^{i-1}Y_1 < b_1\right)P\left(a_2 < k^{i-1}Y_2 < b_2\right)\ldots P\left(a_m < k^{i-1}Y_m < b_m\right) \end{aligned} \tag{2.5}$$

i=1, 2,…,n, since the stresses on the components are independent random variables.

Let $g_j(y)$ be the p.d.f. of Y_j, then

$$R'(i) = \prod_{j=1}^{m} \int_{\frac{a_j}{k^{i-1}}}^{\frac{b_j}{k^{i-1}}} g_j(y)\,dy_j \quad i = 1,2,\ldots,n \tag{2.6}$$

Now from (2.4) and (2.6), we get

$$R(1) = R'(1)$$

$$= \prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)\,dy_j \tag{2.7}$$

$$R(2) = R'(2) - R'(1)R'(2)$$

$$= \prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)\,dy_j - \left(\prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)\,dy_j \right) \left(\prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)\,dy_j \right) \tag{2.8}$$

$$R(3) = R'(3) - R'(2)R'(3) - R'(1)R'(3) + R'(1)R'(2)R'(3)$$

$$= \prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)\,dy_j - \left(\prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)\,dy_j \right) \left(\prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)\,dy_j \right) - \left(\prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)\,dy_j \right)$$

$$\left(\prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)\,dy_j \right) + \left(\prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y)\,dy_j \right) \left(\prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y)\,dy_j \right) \left(\prod_{j=1}^{m} \int_{a_j/k^2}^{b_j/k^2} g_j(y)\,dy_j \right) \tag{2.9}$$

Substituting R (1), R (2),…, R(n) in (2.3) we can obtain R_n.

Stress-Strength follows Specific distribution

In this section, we consider stress and strength variables that follow Rayleigh distributions, which may have different parameters. In the following subsection, we derive the reliability of a 3-cascade system under the model described in Section 2.

Rayleigh stress-strength distribution

Model: Let $g_j(y)$ be the Rayleigh densities with parameters $\beta_j^{\,2}$, j=1, 2,…,m respectively. Then from (2.6) we get,

$$R'(i) = \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k^{i-1}}} - e^{-\frac{b_j}{k^{i-1}}} \right); \quad i = 1,2,\ldots,n \tag{3.1.1}$$

Now from (3.1.1) and using (2.4) we get the marginal reliabilities R(1), R(2) , R(3) as follows.

$$R(1) = R'(1)$$

$$= \prod_{j=1}^{m} \left(e^{-a_j} - e^{-b_j} \right) \tag{3.1.2}$$

$$R(2) = \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) - \prod_{j=1}^{m} \left(e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) \tag{3.1.3}$$

$$R(3) = \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) - \prod_{j=1}^{m} \left(e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) - \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right)$$

$$\prod_{j=1}^{m} \left(e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) + \prod_{j=1}^{m} \left(e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) \prod_{j=1}^{m} \left(e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) \tag{3.1.4}$$

By substituting the values of R (1), R (2) and R (3) from equations (3.1.2) to (3.1.4) into equation (2.3), we obtain R_3.

4. Numerical Evaluation

From the table, we observe that R(i) for i=2,3 increases as k increases. For example, when $a_1 = 0.2,\ b_1 = 3.0,\ a_2 = 0.4,\ b_2 = 3.0,$ then R(1)=0.7189, R(2)=0.1489, R(3)=0.0489. When any of the upper limits increase, R(i) for i=2,3 may decrease in some cases. Similarly, when any of the lower limits increase, R(i) for i=2,3 may also decrease in certain instances.

Table1: Values of R(1), R(2), R(3) and R_3 for the failure model where Stress-Strength Distributions are Rayleigh

a_1	a_2	b_1	b_2	k	R(1)	R(2)	R(3)	R_3
0.2	0.4	3	3	0.4 0.6 1 1.2	0.7189 0.7189 0.7189 0.7189	0.1489 0.1787 0.2164 0.2183	0.0489 0.0566 0.0589 0.0599	0.9167 0.9542 0.9942 0.9971
0.3	0.3	5	5	0.4 0.6 1 1.2	0.6567 0.6567 0.6567 0.6567	0.1561 0.2201 0.2233 0.2243	0.0540 0.0679 0.0722 0.0813	0.8668 0.9447 0.9522 0.9623
1	0.5	6	6	0.4 0.6 1 1.2	0.2898 0.2898 0.2898 0.2898	0.0554 0.1102 0.1347 0.2110	0.0027 0.0383 0.1359 0.1709	0.3479 0.4383 0.5604 0.6717
0.4	0.2	6	6	0.4 0.6 1 1.2	0.4675 0.4675 0.4675 0.4675	0.1211 0.1327 0.2224 0.2437	0.0121 0.0547 0.1763 0.1983	0.6007 0.6549 0.8662 0.9095

References

[1]Doloi. C, Borah. M, and Sriwastav G. L.(2010). Cascade System with Random Attenuation Factor, *IAPQR Transactions*, 35(2), 81-90.

[2] Doloi. C, Gogoi. J, and Pradhani G.(2023). A Cascade System Reliability Model for Rama Distributed Random Variables, *International Journal of Scientific Research in Mathematical and Statistical Sciences*, 10(1), 01-09.

[3] Heller, R.A., and Donat, R.C.(1967). Fatigue Failure of a Redundant Structure, *Structural Fatigue in Aircraft, ASTM STP 404, Amer. Soc. Testing Maths.*, 56-66.

[4] Hilton, H.H., and Fergen, M.(1960). Minimum Weight Analysis Based on Structural reliability, *Jr. of the Aerospace Science*, 27(9).

[5] Moser, F., and Kinser, D.E.(1967). Analysis of Structural Reliability, *Jour. Of Structural Division,* 147.

[6] Pandit, S. N. N. and Sriwastav G. L.(1975). Studies in Cascade Reliability I, *IEEE Trans. On Reliability*, 24(1), 53-56.

[7] Pandit, S. N. N. and Sriwastav G. L.(1978). Studies in Cascade Reliability II, *IEEE Trans. On Reliability*, 27(2), 166-168.

[8] Sriwastav G. L. Dutta, K.(1986). An n-unit Standby Redundant System with Different Types of Failure in S-S Model, Presented at the 73rd Annual Session of ISCA, Jan. 3-8, Delhi, India

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